CSE311 Microwave Engineering

LEC (04)
Electron Motion in Tubes

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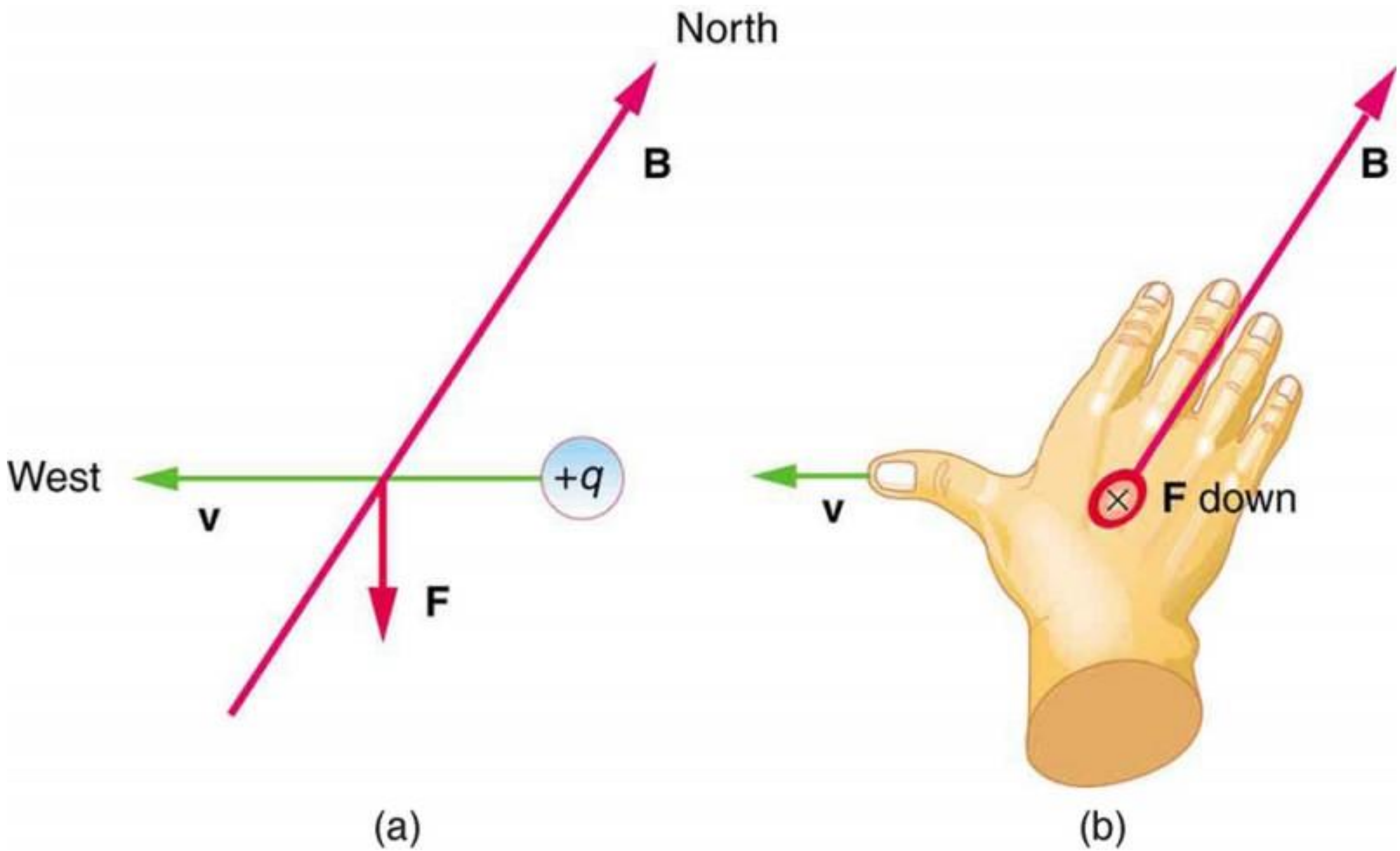
LECTURE OUTLINES

Motion of an electron in Magnetic field only
Trajectory of particle Motion in "B" Field only
Motion of an electron in both electric and Magnetic field
Motion in cylindrical coordinates

Motion of an electron in B fields

- Firstly , assume there is no electric field and there is only magnetic field effect (B)
- Assume that magnetic field is (B), Normal to the page and towards inside.
- If there is no magnetic field, the electron will move in a straight motion. But due to magnetic field it will move in a circular trajectory.
- Due to the magnetic field , the particle (electron / or proton) affected by a force excreted on it due to the magnetic field ($\vec{F}$)
- Using right hand rule (R.H.R)➔ $\vec{F}$ direction can be determined
- Using Lorentz rule we can estimate the force:

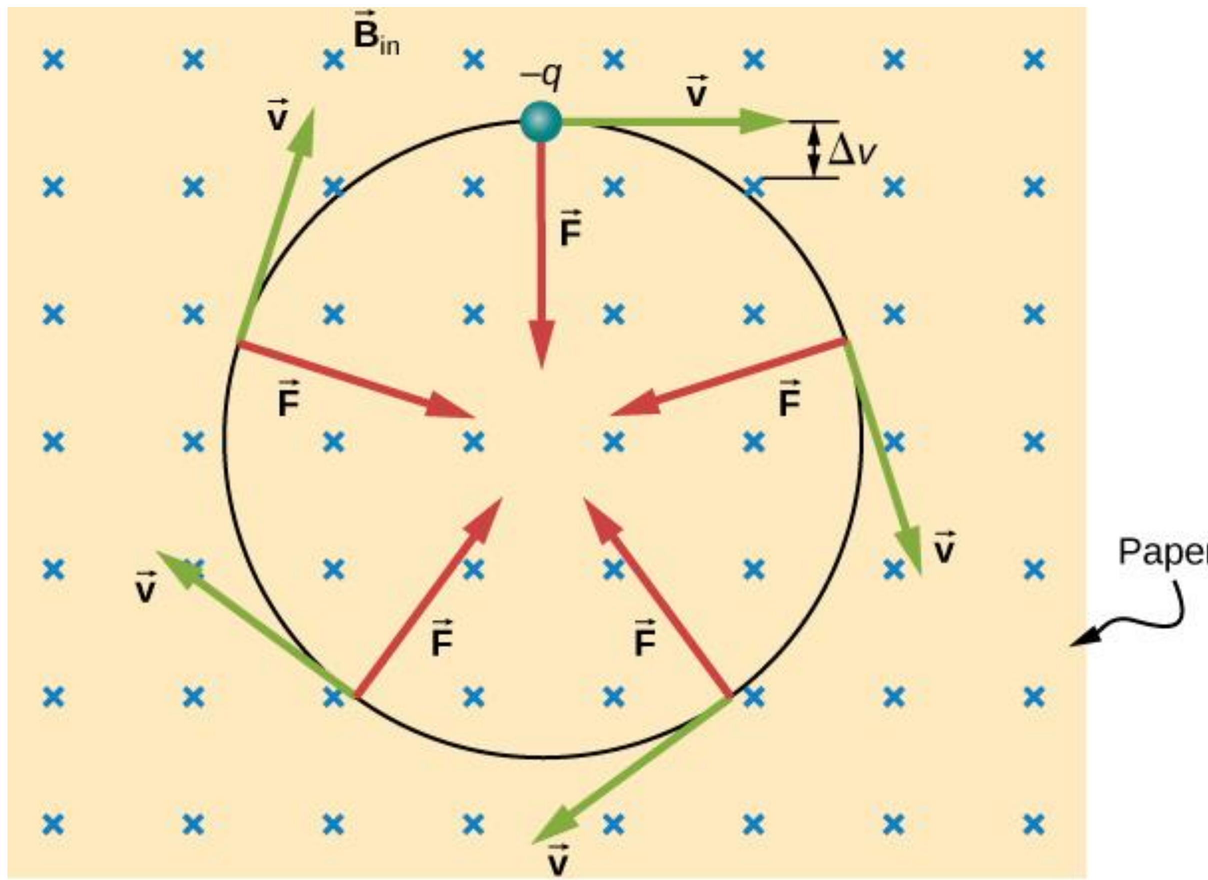
$$\vec{F} = qv \times \vec{B}$$


North
B
West
+q
v
F
B
v
F down
(a)
(b)

- The magnetic force doesn't change the velocity of the moving particle, it only changes the (**direction**).
- The trajectory of the electron/proton is circular motion, so we can estimate the velocity components (v_x, v_y, v_z) at any time.
- It is requires to calculate :
  - The radius of the motion (R)
  - The periodic cyclic time (cyclotron) of the motion (T_p)
- starting from :

$$|\vec{F}| = |qv \times \vec{B}| = qvBsin\theta$$

- At point (1) of the shown figure (expected circular motion)
  - $\theta = 90^o$,
  - $|\vec{F}| = qvBsin\theta = qvBsin90^o = qvB$
  - But $|\vec{F}| = \frac{mv^2}{R}$
  - So , $qvB = \frac{mv^2}{R}$



Now :

- ***Raduis of Cicle*** $(\boldsymbol{R}) = \dfrac{mv}{qB}$

- ***Periodic Time*** $(\boldsymbol{T_p}) = \dfrac{distance}{velocity}$

$$= \frac{2\pi R}{v}$$

$$= \frac{2\pi}{v} \times \frac{mv}{qB}$$

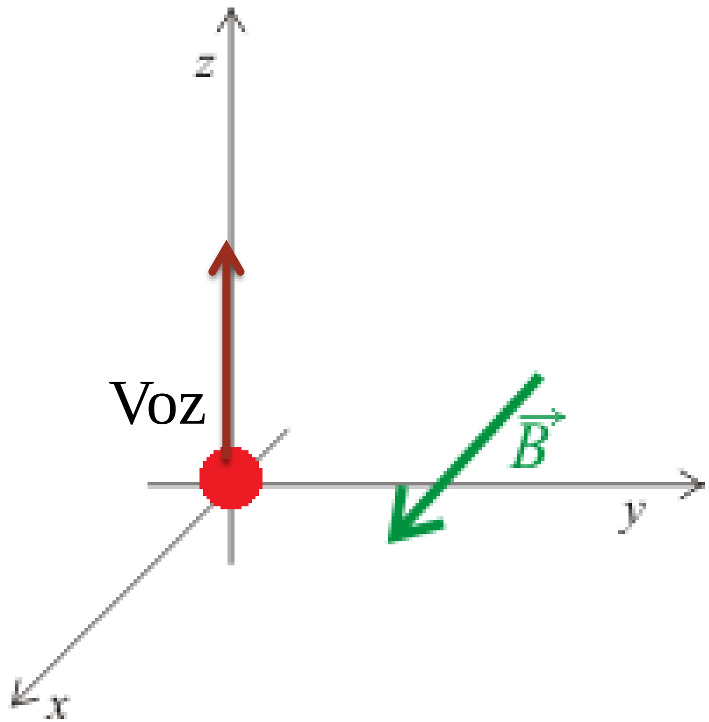
$$(\boldsymbol{T_p}) = \frac{2\pi m}{qB}$$

Trajectory of particle Motion in “B” Field only

Case (1) :

- NO Electric field
- The magnetic field is in $\vec{x}$ direction only
- The initial velocity is in $\vec{z}$ direction

- Firstly , assume that the magnetic field is in $\vec{x}$ direction only and the initial velocity is in $\vec{z}$ direction



- *Assume* $v_{ox} = v_{oy} = 0$, $v_{oz} > 0$

- *Assume initial position is at origin* $x_o = y_o = z_o$

- $velocity\ vector\ \vec{v} = v_x\overrightarrow{a_x} + v_y\overrightarrow{a_y} + v_z\overrightarrow{a_z}$

- $\vec{F} = q(v_x\overrightarrow{a_x} + v_y\overrightarrow{a_y} + v_z\overrightarrow{a_z}) \times (B_x\overrightarrow{a_x})$

$$\vec{F} = \begin{vmatrix} \overrightarrow{a_x} & \overrightarrow{a_y} & \overrightarrow{a_z} \\ v_x & v_y & v_z \\ B_x & 0 & 0 \end{vmatrix}$$

$$\vec{F} = q(B_x v_z\overrightarrow{a_y} - B_x v_y\overrightarrow{a_z})$$

$$\vec{F} = m\frac{dv_x}{dx}\overrightarrow{a_x} + m\frac{dv_y}{dy}\overrightarrow{a_y} + m\frac{dv_z}{dz}\overrightarrow{a_z}$$

$$m\frac{dv_x}{dx} = 0$$

$$v_x = \int \frac{dv_x}{dx}\, dx$$

v_x = Constant

At t = 0 , v_{ox} = 0 , so const. = 0 and

so $\boxed{v_x = 0}$

$$x = \int v_x dx = \text{constant}$$

At t = 0 , x_o = 0 , so const. = 0 and

$$\boxed{x = 0}$$

$$m\frac{dv_y}{dy} = qB_x v_z \rightarrow (1)$$

$$m\frac{dv_z}{dz} = -qB_x v_y \rightarrow (2)$$

- ❑ Differentiate (1)

$$m\frac{d^2v_y}{dt^2} = qB_x\frac{dv_z}{dz}$$

- ❑ Substitute in (2)

$$m\frac{d^2v_y}{dt^2} = qB_x(-\frac{q}{m}B_x v_y)$$

$$\frac{d^2 v_y}{dt^2} + \frac{q^2 {B_x}^2}{m^2} v_y = 0$$

$$\frac{d^2 v_y}{dt^2} + \omega^2 v_y = 0 \Rightarrow \omega^2 = \frac{q^2 {B_x}^2}{m^2} \text{ or } \omega = \frac{q B_x}{m}$$

It is a differential equation, has a solution of :

v_y(t) = A sin ωt + D Cos ωt

➔ At t = 0 , v_{oy}(t)=0 && D=0

➔ So v_y(t) = A sin ωt

$v_y(\mathrm{t}) = \mathrm{A}\sin\omega\mathrm{t}$

By differentiation :

$$\frac{dv_y(\mathrm{t})}{dt} = \omega\,\mathrm{A}\cos\omega\mathrm{t}$$

$$we\ know\ that\quad m\frac{dv_y}{dy} = qB_x v_z\ \&\&\ \omega = \frac{qB_x}{m}$$

$$m\left(\frac{qB_x}{m}\right)\mathrm{A}\cos\omega\mathrm{t} = qB_x v_z$$

So $\mathrm{A}\cos\omega\mathrm{t} = v_z$

$A \cos \omega t = v_z$

Now , at t = 0 , $A = v_{oz}$

So , $v_y(t) = v_{oz} \sin \omega t$

➔ To calculate v_z

we know that $m\dfrac{dv_y}{dy} = qB_x v_z$

$m(\omega v_{oz} \cos \omega t) = qB_x v_z$

$$v_z = \frac{m\omega v_{oz} \cos \omega t}{qB_x} = \frac{m\frac{qB_x}{m} v_{oz} \cos \omega t}{qB_x} = v_{oz} \cos \omega t$$

$$x = 0$$

$$v_x=0$$

$$v_y(\text{t}) = v_{oz} \sin \omega \text{t}$$

$$v_z = v_{oz} \cos \omega \text{t}$$

Now we wanna calculate Z and y

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$v_y = \frac{dy}{dt}$ $\qquad$ $y = \int v_y \, dt$

$$y = \int v_{oz} \sin \omega t \; dt = \frac{-v_{oz}}{\omega} \cos \omega t + C$$

At t = 0 , y_o = 0 , and

$$0 = \frac{-v_{oz}}{\omega} \cos 0 + C \rightarrow C = \frac{v_{oz}}{\omega}$$

$$y = \int v_{oz} \sin \omega t \; dt = \frac{-v_{oz}}{\omega} \cos \omega t + \frac{v_{oz}}{\omega} = \frac{v_{oz}}{\omega} (1 - \cos \omega t)$$

Or it can be re-written

$$y - \frac{v_{oz}}{\omega} = \frac{-v_{oz}}{\omega} \cos \omega t \rightarrow \text{(I)}$$

Now we wanna calculate Z and y

$v_z = \frac{dz}{dt}$ $\qquad z = \int v_z \, dt$

$$z = \int v_{oz} \text{ Cos } \omega t \; dt = \frac{v_{oz}}{\omega} \sin \omega t + C$$

At t = 0 , $z_o = 0$, and

$$0 = \frac{v_{oz}}{\omega} \sin 0 + C \Rightarrow C = 0$$

$$z = \int v_{oz} \cos \omega t \; dt = \frac{v_{oz}}{\omega} \sin \omega t$$

$$z = \frac{v_{oz}}{\omega} \sin \omega t \qquad y - \frac{v_{oz}}{\omega} = \frac{-v_{oz}}{\omega} \cos \omega t$$

$$\left(y - \frac{v_{oz}}{\omega}\right)^2 + z^2 = \left(\frac{-v_{oz}}{\omega} \cos \omega t\right)^2 + \left(\frac{v_{oz}}{\omega} \sin \omega t\right)^2 = \left(\frac{v_{oz}}{\omega}\right)^2$$

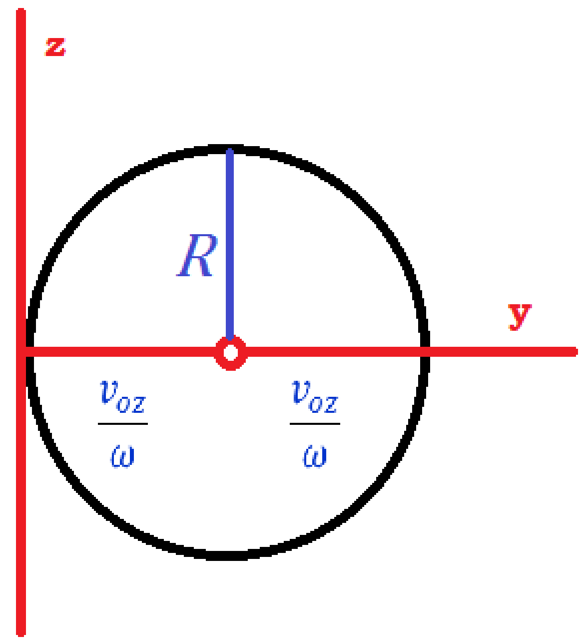
$$\Longrightarrow \left(y - \frac{v_{oz}}{\omega}\right)^2 + z^2 = \left(\frac{v_{oz}}{\omega}\right)^2$$

So the trajectory is circular motion with radius $\frac{v_{oz}}{\omega}$

$$R = \frac{v_{oz}}{\omega} = \frac{v_{oz}}{\frac{qB_x}{m}}$$

$$R = \frac{mv_{oz}}{qB_x}$$

$$T = \frac{2\pi R}{v_{oz}} = \frac{2\pi \frac{mv_{oz}}{qB_x}}{v_{oz}} = \frac{2\pi m}{qB_x}$$



Case (2) :

- NO Electric field
- The magnetic field is in $\vec{x}$ direction only
- The initial velocity is in $\vec{x}$ and $\vec{z}$ direction

- Do it Yourself to reach :

$$v_y = v_{oz} \sin \omega t$$

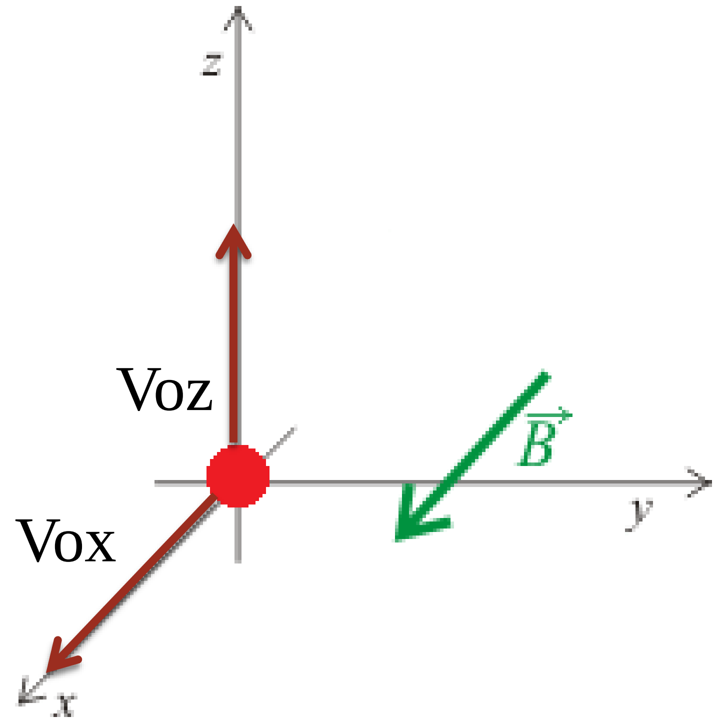
$$v_z = v_{oz} \cos \omega t$$

$$v_x = v_{ox}$$

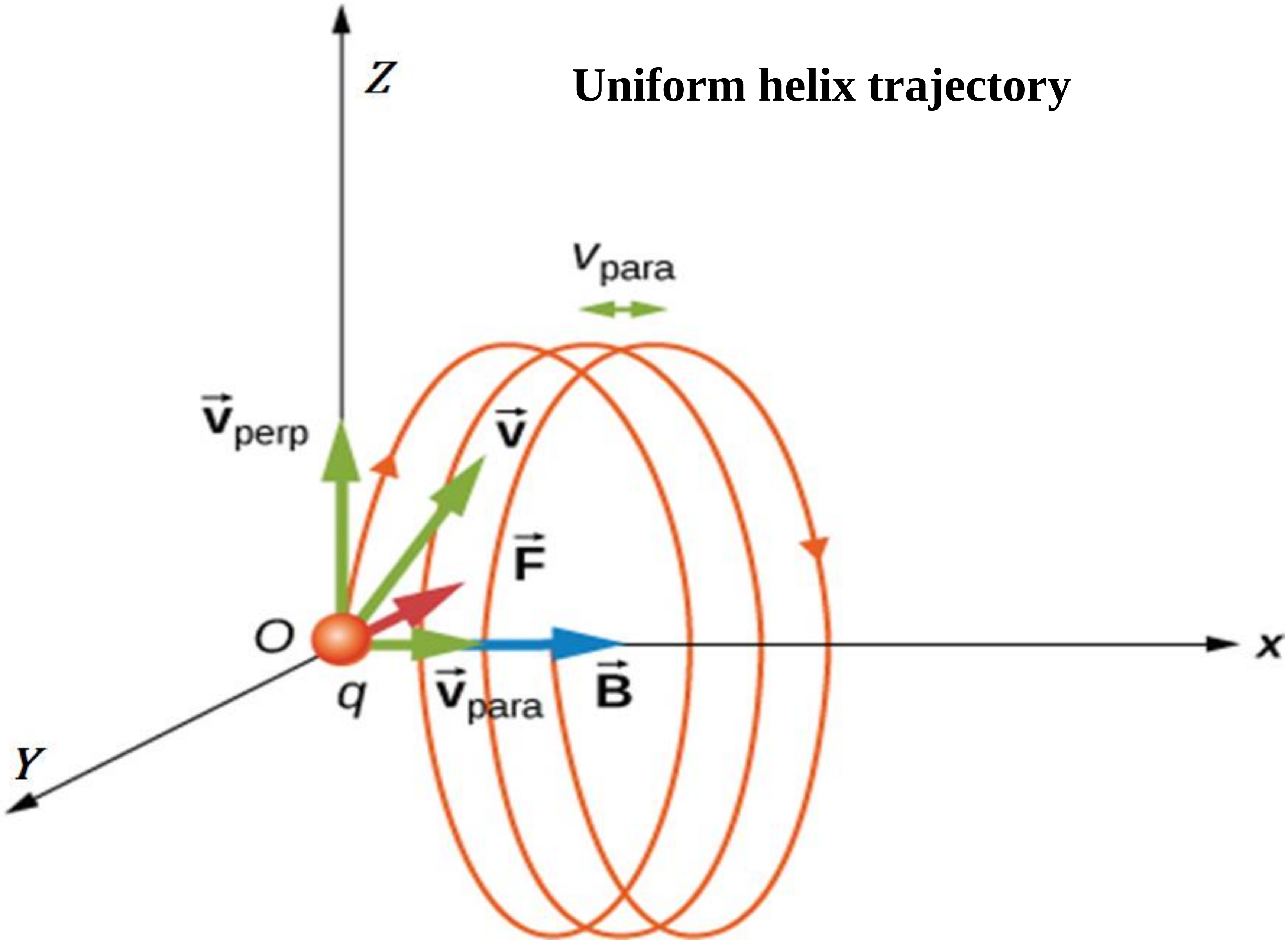
$$x = v_{ox} t$$

$$z = \frac{v_{oz}}{\omega} \sin \omega t$$

$$y - \frac{v_{oz}}{\omega} = \frac{-v_{oz}}{\omega} \cos \omega t$$



Uniform helix trajectory



Case (3)

- Uniform Electric field $\vec{x}$ direction only
- The magnetic field is in $\vec{x}$ direction only
- The initial velocity is in $\vec{z}$ direction

- Do it Yourself to reach :

$$v_y = v_{oz} \sin \omega t$$

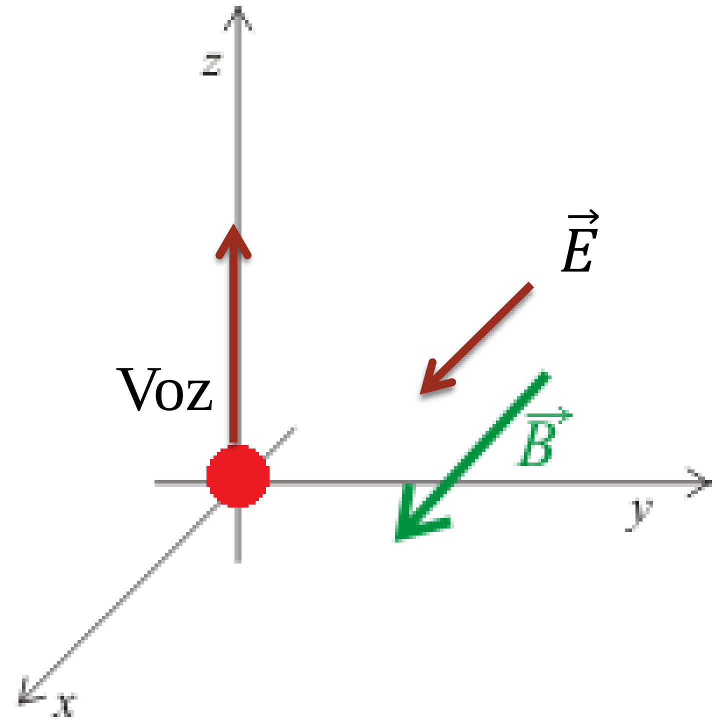
$$v_z = v_{oz} \cos \omega t$$

$$v_x = \frac{qE}{m} t$$

$$x = \frac{qE}{2m} t^2$$

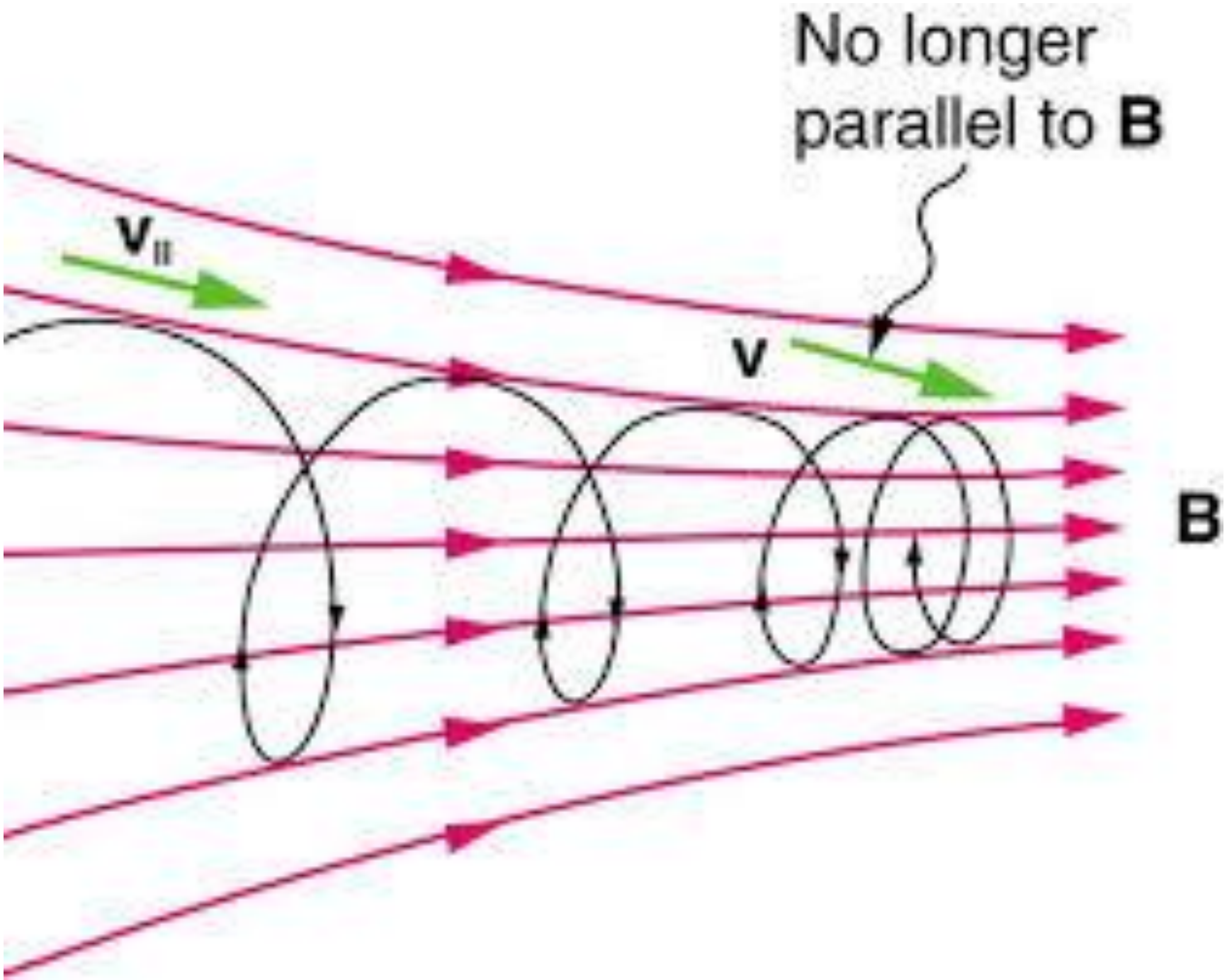
$$z = \frac{v_{oz}}{\omega} \sin \omega t$$

$$y - \frac{v_{oz}}{\omega} = \frac{-v_{oz}}{\omega} \cos \omega t$$



$$\vec{F} = q(\vec{E} + v \times \vec{B})$$

Non uniform helix trajectory



Case (4)

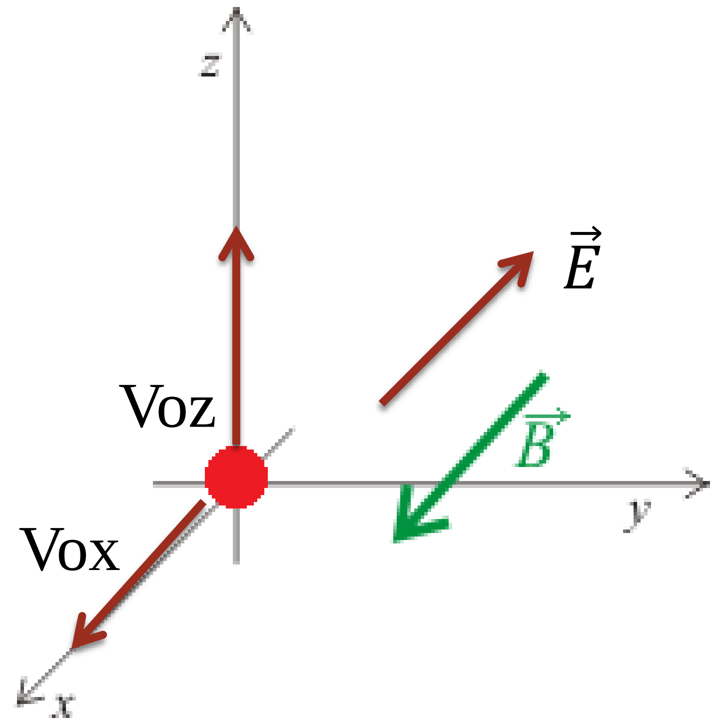
- Uniform Electric field $\overrightarrow{-x}$ direction only
- The magnetic field is in $\vec{x}$ direction only
- The initial velocity is in $\vec{x}$ ***and*** $\vec{z}$ direction

- Do it Yourself to reach :

$$v_z = v_{oz} \cos \omega t$$

$$v_x = \frac{-qE}{m} t + v_{ox}$$

$$v_y = v_{oz} \sin \omega t$$



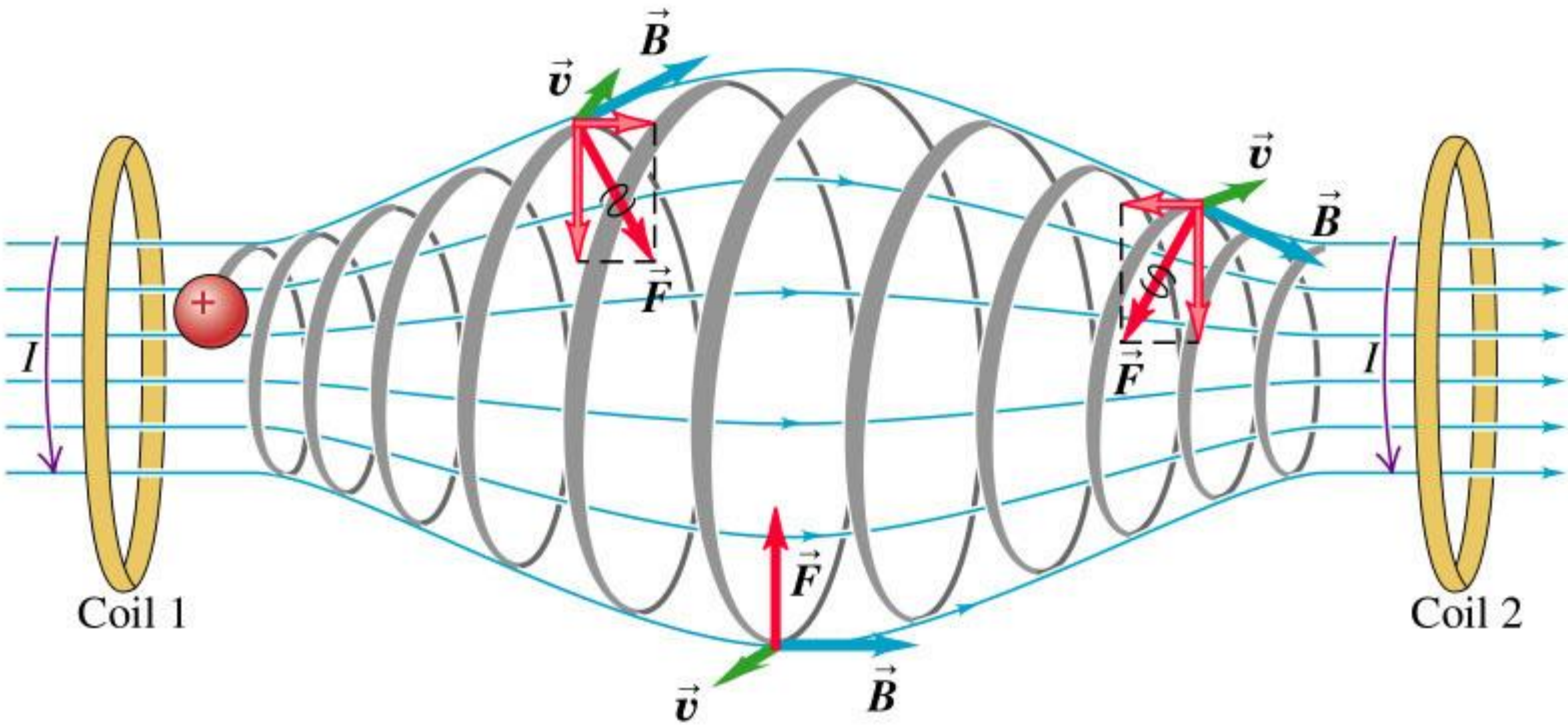
$$\vec{F} = q(\vec{E} + v \times \vec{B})$$

$$x = \frac{-qE}{2m} t^2 + v_{ox} t$$

$$z = \frac{v_{oz}}{\omega} \sin \omega t$$

$$y - \frac{v_{oz}}{\omega} = \frac{-v_{oz}}{\omega} \cos \omega t$$

Helical Motion in X and circular in YZ



Project

Thank you for your attention